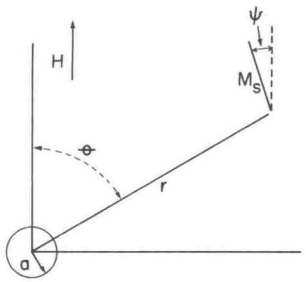

FIG. 1. Spherical cavity of radius a. $M_s \cos\psi$ is the magnetization in the direction of the applied field at the spherical coordinate (r, θ, Φ). There is azimuthal symmetry.

are greater than 10^{-3}.

The magneto-elastic energy expression for the isotropic material is obtained by averaging the single-crystal expression

$$E_{me} = b_1(\alpha_1^2 e_{11} + \alpha_2^2 e_{22} + \alpha_3^2 e_{33}) + 2b_2(\alpha_1\alpha_2 e_{12} + \alpha_2\alpha_3 e_{23} + \alpha_2\alpha_1 e_{21}) \quad (5)$$

over random crystal orientation using Eq. (4) for the strain components.[12] This average assumes fixed strains with magnetization in fixed axis. The result is

$$E_{me} = \frac{3}{4}\frac{BP}{\mu}\frac{a^3}{r^3}\cos^2(\psi+\theta)\,, \quad (6)$$

where $B = \frac{2}{5}b_1 + \frac{3}{5}b_2$. The angles are shown in Fig. 1.

Justification for ignoring the remaining energy terms in Eq. (3) is as follows: Consider the crystalline anisotropy energy first. Since the terms in the total energy expression are additive, the contribution of the anisotropy energy to the magnetization in the approach to saturation region may be superimposed. This was done by Parfenov and Voroshilov using the expression

$$\frac{M}{M_s} = 1 - \frac{8}{105}\frac{K^2}{H^2 M_s^2} + \cdots \quad (7)$$

derived by Akulov.[1] This will not be done in the present work. Although this refinement would improve the calculation, it tends to obscure the primary objective of the paper.

The exchange energy is commonly expressed in the form

$$E_{ex} A[(\vec{\nabla}\alpha_1)^2 + (\vec{\nabla}\alpha_2)^2 + (\vec{\nabla}\alpha_3)^2]\,,$$

where A is the exchange constant. The exchange energy will be on the order of A/a^2, where a is the dimension of the spherical cavity. A rough estimate for YIG from molecular field theory is $A \simeq 3\times10^{-7}$ erg/cm. The cavity dimension is approximately 1 μ.[6,9] This gives an exchange energy on the order of 30 erg/cm^3. The magneto-elastic energy for YIG is on the order of $(\frac{3}{4})BP/\mu$. At $P = 10$ kbar this is 5×10^4 erg/cm^3, which is better than three orders of magnitude larger than the exchange energy.

The demagnetizing energy is difficult to assess. The worst imaginable case is a spherical cavity in an otherwise uniform infinite magnetization field. The demagnetizing energy associated with this is

$$E_d = \tfrac{2}{3}\pi M_s^2 (a^3/r^3)[3\cos^2\theta - 1]\,.$$

For YIG this energy is about 10^4 erg/cm^3, which is about 20% of the magneto-elastic energy. In practice this is much too high. Such a drastic difference between the exchange and demagnetizing energy would not occur since some form of domain structure would occur in order to reduce this difference.

The justification for ignoring the remaining energy terms is not intended to be rigorous. It does suggest that, in the porous material problem, and in the case of severe internal strain, the magneto-elastic energy is an extremely significant, if not dominant, term. Since this work intends to filter out one term (the magneto-elastic energy) as responsible for the a/H behavior in many magnetic materials, the remaining terms will not be considered.

The surviving energy expression includes the magneto-elastic energy and the interaction energy:

$$E = \frac{3}{4}\frac{BP}{\mu}\frac{a^3}{r^3}\cos^2(\psi+\theta) - HM_s\cos\psi\,. \quad (8)$$

In magnetic equilibrium the variation of E with respect to the coordinate ψ must be a minimum. This gives

$$\frac{3}{4}\frac{B}{\mu M_s}\frac{P}{H}\frac{a^3}{r^3}\sin 2(\psi+\theta) - \sin\psi = 0\,. \quad (9)$$

In order to relate Eq. (9) to the macroscopic magnetization of the porous material, the component of magnetization in the direction of the applied field, $M_s\cos\psi$, must be solved for and integrated over a volume determined by the porosity of the material. This is difficult in practice since Eq. (9), when unfolded in terms of $\cos\psi$, yields a quartic equation. This problem is considered analytically in Sec. III and solved numerically in Sec. IV.

III. ANALYTIC CONSIDERATIONS

Since interest lies in the approach to saturation, a series solution about the point $P/H = 0$, if sufficiently well behaved, would be of value. A solution of this form is possible. Although $\cos\psi$ cannot be solved for explicitly in Eq. (9), implicit derivatives of all orders can be obtained and solved for when evaluated at the point of expansion. The series takes the form

$$\frac{M}{M_s}=\frac{1}{v}\int_v \left(1+\frac{\partial\cos\psi}{\partial(P/H)}\bigg|_0 \frac{P}{H}+\frac{1}{2}\frac{\partial\cos\psi}{\partial(P/H)^2}\bigg|_0 \frac{P}{H}^2+\cdots\right)dV\,, \tag{10}$$

where the spherical volume of integration is determined by the porosity of the material. When evaluated, the first few terms of the series are

$$\frac{M}{M_s}=1-\frac{3}{20}\left(\frac{BP}{\mu M_s H}\right)^2 p+\frac{9}{280}\left(\frac{BP}{\mu M_s H}\right)^3 p + \frac{27}{12\,320}\left(\frac{BP}{\mu M_s H}\right)^5 p+\cdots\,, \tag{11}$$

where p is the porosity of the material. This series predicts the magnetization curve for the porous material in the approach to saturation region. Explicit dependence on external pressure, applied field, and porosity are shown. It is worth noting that a linear term does not appear.

Comparison of this series with the Becker-Polley expression

$$\frac{M}{M_s}=1-\frac{3}{5}\frac{\lambda_s^2\langle\sigma_i^2\rangle_{av}}{M_s^2H^2}\,, \tag{12}$$

where $\lambda_s=-B/3\mu$ in a magnetically isotropic medium, shows that they are quite similar in a region where the series can be approximated by the first nonvanishing term. This is mentioned for a comparison of the previously predicted effect of internal strain on one hand [see Eq. (2)] and the effect of induced strain in the present problem on the other.

The behavior of this series can best be shown by considering a particular example. With the properties of YIG and a representative value of $P/H=0.1$, which was chosen from values of strain and applied field used in the work of Ref. 6, the resulting first few terms in the series are

$$M/M_s=1-0.032+0.041+0.068+\cdots\,.$$

This serves only to illustrate that the functional dependence of M/M_s on P/H in this region is not well represented by the first few terms in the series. In other words, the series does not converge sufficiently fast in this region of the magnetization curve to make its use worthwhile.

It can be shown that the subsequent functional dependence of M/M_s on P/H, following the initial quadratic behavior predicted by Eq. (11), is linear and has a slope given by

$$\frac{dM/M_s}{dP/H}=-\gamma\frac{B}{\mu M_s}p\,, \tag{13}$$

where γ is a constant independent of material properties. This can be shown analytically, but the calculation represents a last-effort attempt to circumvent the computer. Since a computer solution was ultimately required, this calculation is relegated to the Appendix. This calculation is important, however, since it predicts the linear behavior in a limited region, whereas the numerical solution only strongly suggests it. A glance forward to the numerical solutions in Fig. 2 may help clarify the results of this section.

IV. NUMERICAL SOLUTION

The equilibrium relation (9) was solved for $\cos\psi$ and this term integrated over the required volume by conventional numerical techniques. Solutions for various porosities are shown in Fig. 2. The following observations are noted.

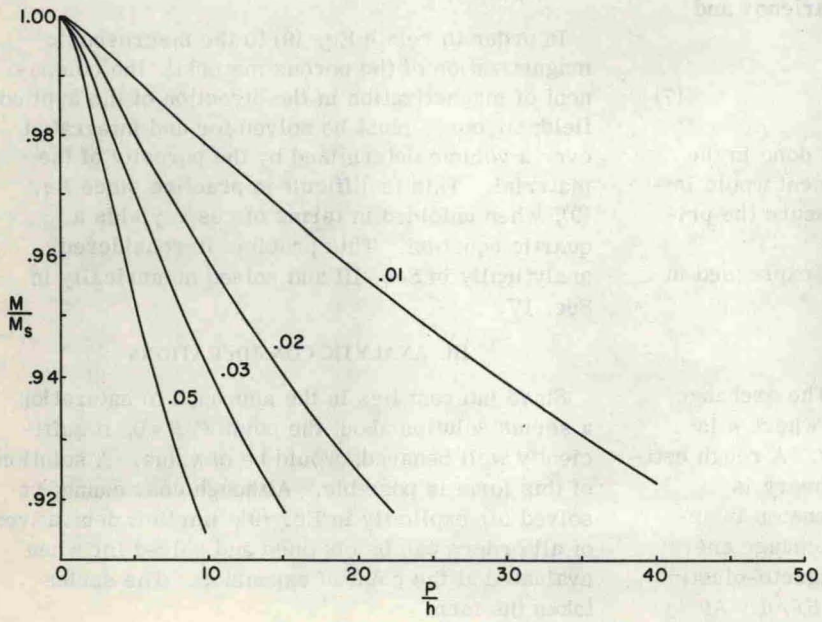

FIG. 2. Numerical solution of pressure-induced deviation from saturation magnetization. $h=H/(\mu M_s/B)$ is the reduced field. Solutions are for porosities of 0.01, 0.02, 0.03, and 0.05. An estimate of the normalized slope, γ = slope/porosity, was approximately $\gamma=0.21$ for the four curves.